

Distributed Energy and Reserve Scheduling in Local Energy Communities Using L-BFGS Optimization

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Abstract—Encouraging citizens to invest in small-scale renewable resources is crucial for transitioning towards a sustainable and clean energy system. Local energy communities (LECs) are expected to play a vital role in this context. However, energy scheduling in LECs presents various challenges, including the preservation of customer privacy, adherence to distribution network constraints, and the management of computational burdens. This paper introduces a novel approach for energy scheduling in renewable-based LECs using a decentralized optimization method. The proposed approach uses the Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) method, significantly reducing the computational effort required for solving the mixed integer programming (MIP) problem. It incorporates network constraints, evaluates energy losses, and enables community participants to provide ancillary services like a regulation reserve to the grid utility. To assess its robustness and efficiency, the proposed approach is tested on an 84-bus radial distribution network. Results indicate that the proposed distributed approach not only matches the accuracy of the corresponding centralized model but also exhibits scalability and preserves participant privacy.

Index Terms—Distributed optimization, flexibility services, L-BFGS method, local energy community, renewables, reserve.

I. INTRODUCTION

LOCAL energy communities (LECs), formed to locally integrate small-scale energy resources and consumers, promote sustainable and clean energy systems [1], [2]. These communities harness renewable resources like photovoltaics and energy storage systems, not only contributing to reduced pollutants but also enabling direct power exchanges among participants. Such exchanges offer economic advantages by lowering energy procurement costs and optimizing resource

utilization, defining the efficient use of the available resources. The end customers who are equipped with small-scale power generation resources, called prosumers from now on, can sell back energy to the grid or other community participants. Therefore, the concept of peer-to-peer (P2P) energy trading is a key characteristic of the LEC concept [3].

Most of the previous studies focused on virtual P2P energy trading between prosumers without considering the effects of the energy exchanges on the network operation [4]. Nevertheless, the constraints of the power grid and the power losses should be considered in the modeling of the LEC scheduling.

In addition to the above-mentioned aspects, LECs are expected to provide ancillary and flexibility services to the utility grid. However, the high number of actors and setting variables lead to a complex modelling representation as well as a higher computational burden of the problem [5].

A review of the relevant literature shows various approaches. In [6], energy transaction scheduling in LECs using a P2P model and mixed-integer linear programming is explored, emphasizing cost reduction but neglecting ancillary services and power flow constraints. Corporate game theory is applied in P2P energy trading in [7], while [8] discusses P2P trading in Nepal's electricity market without addressing distribution network constraints. In [9], a method for optimal management of capacity using a blockchain-based P2P in low-voltage distribution networks has been presented. Regardless of the ancillary service market, the method allows keeping the voltage within the adequate range by providing flexibility to different nodes of the network.

However, implementing P2P energy transactions in LECs present challenges, particularly in preserving user privacy [10]. Various models of LEC integration within the power distribution network are reviewed in [11]. Although the grid layer has been neglected in [12], [13], power losses, voltage limitations and power flow constraints have been considered by other methods for the LEC energy scheduling. Notably, in [14], the cost minimization of energy procurement in the LEC is achieved by a distributed algorithm based on the alternating direction method of multipliers (ADMM), in which network power losses are allocated to each energy transaction between prosumers and to the energy transactions between a prosumer and the main grid. The procedure is extended in [15] including a day-ahead stochastic optimization and intraday scheduling based on rolling horizon optimizations. In [16], a linearized model of AC power flow was adopted to reduce the computational burden of the LEC market clearing problem using

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two-stage stochastic programming. The linearized distribution power flow method requires less information on the network characteristics with a limited reduction in accuracy [17], [18]. To consider network constraints in community energy trading, two web services for electricity market clearing and power flow analysis have been presented in [19].

As mentioned, LECs are also expected to provide significant flexibility services for power system operation and control. Flexibility is enhanced through the adoption of energy storage systems, electric vehicle charging management, the implementation of demand response programs, and innovative methods like electricity-to-heat conversion, compressed air, or hydrogen. Furthermore, the capability of photovoltaic converters to provide reactive power can be used for the optimization of the voltage profile [20]. Research in European countries shows that the flexibility provided by medium-scale energy communities reduces the costs of heat and electricity systems and postpones the generation expansion needs [21]. In [22], the energy management in the LEC is based on the use of the Markov modeling method and reinforcement learning technique, showing that trading energy in the form of a local energy pool increases prosumers' income and surplus with more efficient use of renewable energy.

In the analysis presented in [23], frequency restoration reserve as balance services to the transmission network operator is achieved through the aggregation of prosumers. Due to the large number of prosumers, a distributed method has been used, showing adequate scalability. The mechanism for providing ancillary services presented in [24] aims at maximizing the benefit of end customers via P2P energy trading, without considering network constraints. The centralized day-ahead energy and reserve scheduling of energy resources in the LEC presented in [25] addresses the uncertainties related to electricity price and renewable energy power through a robust optimization method for the scheduling of the spinning reserve. Two-stage stochastic programming in a centralized market framework is proposed in [26] for the analysis of the reactive power provision by distributed energy resources as an ancillary service. Using flexible energy resources like electric vehicles and batteries in an LEC, a centralized approach without considering network constraints is adopted in [27] for the day ahead and real-time operational scheduling and the provision of frequency restoration reserve. In [28], a stochastic method for energy and reserve scheduling of a large number of PV-battery units is used for the assessment of the participation of aggregators and end customers in the ancillary service market. Distributed parallel optimization is adopted in [29] for the solution of a day ahead and real-time ancillary service market clearing mechanism, to assure the scalability for a large number of prosumers equipped with PV-battery systems.

Considering the approaches and results already presented in the literature on the subject, decentralized methods are preferable to achieve the scalability of the problem related to up/down reserve allocation provided by a large number of prosumers and to preserve privacy [4]. However, incorporating network constraints into decentralized methods requires further study.

This work builds upon the authors' previous research [30], in which an optimal conic projection method based on local computations has been presented to improve prosumers' privacy preservation. Unlike common distributed methods, such as ADMM, which rely on off-the-shelf solvers like GUROBI [31] and [32], CPLEX [33] or MOSEK [34] to solve the distributed optimization problem, the method presented in [30] did not require the use of such solvers. However, [30] focused only on energy scheduling and did not incorporate ancillary services into the model. This paper introduces the simultaneous energy and reserve scheduling of an LEC using a distributed approach.

Moreover, in this paper, the use of the Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) algorithm to solve linear programs in mixed-integer programming is analyzed. This approach enables the modeling of energy scheduling optimization, including up-regulation reserve as an ancillary service. In contrast to the framework presented in [4], which relies on P2P exchanges based on the virtual model of self-consumption, the model presented in this paper is characterized by non-virtual P2P exchanges, with the assessment of network constraints and power losses.

To the best of our knowledge, so far, the simultaneous energy and reserve scheduling in the LEC using a non-centralized framework and based on the P2P energy transactions between prosumers has not been fully addressed in the literature. Accordingly, the main contributions of this paper are summarized as follows.

- A simultaneous energy and reserve scheduling of an LEC using a distributed approach that considers network constraints and power losses is proposed.
- A unified approach utilizing the Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS) algorithm to solve linear programs in the original Mixed-Integer Programming (MIP) problem is proposed. This approach effectively addresses non-convexities without requiring complex specialized optimization algorithms. Moreover, it transforms the MIP problem into downstream sub-problems, eliminating the need for computationally intensive MIP solvers. Indeed, it allows for the use of faster and more scalable quadratic programming (QP) solvers, significantly improving computational efficiency and scalability in large optimization scenarios.
- The dual variables associated with power equilibrium constraints are used to evaluate the prices of internal peer-to-peer (P2P) transactions.

This paper is organized as follows. Section II describes the energy and reserve scheduling formulations. Section III presents the mathematical formulation of the optimization method. Section IV is devoted to the simulation results, demonstrating the performance of the method. Finally, the conclusions are drawn in Section V.

II. MODEL FORMULATION

An LEC consists of several prosumers (or simply producers and consumers) connected to the same distribution network.

The network configuration is radial. All the prosumers connected to a node are aggregated. Each prosumer consists of electric loads, generation units (in this paper we assume photovoltaic units) and battery storage units. Each prosumer can receive energy from the main grid or sell its excess energy to it. Furthermore, prosumers are allowed to exchange energy with each other. With this definition, the objective function of the problem is to minimize the cost of buying energy from the main grid. Thus, the total cost of net energy transaction between LEC and the main grid is given as follows.

$$C_{\text{grid}} = \sum_{s \in S} \pi_s \left\{ \sum_{i \in N} \sum_{t \in T} (\pi_{\text{buy},t} P_{\text{buy_grid},i,t,s} - \pi_{\text{sell},t} P_{\text{sell_grid},i,t,s}) \Delta t \right\} \quad (1)$$

where i and t denote the prosumers and time interval indices, respectively; s is the scenario from the number of possible scenarios S and π_s is the probability of occurrence of a scenario s . $\pi_{\text{buy},t}$ and $\pi_{\text{sell},t}$ represent the electricity prices for buying or selling energy from/to the main grid, respectively. $P_{\text{buy_grid},i,t,s}$ and $P_{\text{sell_grid},i,t,s}$ are the power purchased and sold from/to the main grid. The cost is calculated for the optimization horizon T with each time interval duration of Δt . Generally, the price of buying energy is higher than the price of selling energy. For this reason, prosumers or generally the LEC prefer to maximize the energy internal balance.

The flexibility provided by the LEC is the amount of regulation reserve offered by prosumers.

$$R_{\text{flex}} = \sum_{s \in S} \pi_s \left\{ Bid_{\text{UP}} \sum_{t_{\text{UP}}^i} \left[\sum_{i \in N} P_{i,t,s}^{\text{base}} - \sum_{i \in N} P_{\text{grid},i,t,s} \right] + Bid_{\text{DOWN}} \sum_{t_{\text{D}}^i} \left[\sum_{i \in N} P_{\text{grid},i,t,s} - \sum_{i \in N} P_{i,t,s}^{\text{base}} \right] \right\} \Delta t \quad (2)$$

where Bid_{UP} and Bid_{DOWN} represent the price offer for providing up and down regulation reserve, respectively; $P_{i,t,s}^{\text{base}}$ is the baseline power profile of each prosumer.

Although total operating costs should be reduced, flexibility must be maximized. Therefore, the cost function is defined as follows.

$$\min(C_{\text{grid}} - R_{\text{flex}}) \quad (3)$$

In the proposed model, the cost objective function is augmented by adding the penalty coefficients term. This penalization is applied to control the variables of network losses, battery charge and discharge losses and avoid the reselling of grid energy by the LEC. Therefore, the augmented objective function is presented as (4).

$$OF = \min \left[\sum_{s \in S} \pi_s \left\{ \sum_{i \in N} \sum_{t \in T} (\pi_{\text{buy},t} P_{\text{buy_grid},i,t,s} - \pi_{\text{sell},t} P_{\text{sell_grid},i,t,s}) - Bid_{\text{UP}} \sum_{t_{\text{UP}}^i} \left[\sum_{i \in N} P_{i,t,s}^{\text{base}} - \sum_{i \in N} P_{\text{grid},i,t,s} \right] \right. \right. \right.$$

$$\left. - Bid_{\text{DOWN}} \sum_{t_{\text{D}}^i} \left[\sum_{i \in N} P_{\text{grid},i,t,s} - \sum_{i \in N} P_{i,t,s}^{\text{base}} \right] + \sum_{s \in S} \sum_{i \in N} \sum_{t \in T} (\mu_{\text{loss},i} r_i u_{i,t,s} + \mu_{\text{BES},i} l_{\text{charge},i,t,s} + \mu_{\text{BES},i} l_{\text{discharge},i,t,s} + \mu_{P_{\text{LEC}}}) \hat{P}_{\text{LEC},i,t,s} \right\} \Delta t \quad (4)$$

where μ_{loss} , μ_{BES} and $\mu_{P_{\text{LEC}}}$ represent the penalization coefficients related to the line power loss, charge and discharge battery energy storages (BESs) losses, and the power exchanges with the other prosumers, respectively; $l_{\text{charge},i,t,s}$ and $l_{\text{discharge},i,t,s}$ are power losses during the charging and discharging process of BESs, respectively; $\hat{P}_{\text{LEC},i,t,s}$ is a nonnegative variable defined to be penalized by the objective function to prevent the prosumers from reselling electricity from the grid. The power exchange of prosumer i at period t and scenario s ($P_{\text{LEC},i,t,s}$) is limited by this penalization term as given in (5).

$$P_{\text{LEC},i,t,s} - \hat{P}_{\text{LEC},i,t,s} \leq 0; \quad -P_{\text{LEC},i,t,s} - \hat{P}_{\text{LEC},i,t,s} \leq 0 \quad (5)$$

The energy storage devices used in the grid are of the battery type, whose relationships are given below.

$$P_{\text{BES},i,t,s} - P_{\text{BES},i,t,s}^+ + P_{\text{BES},i,t,s}^- = 0 \quad (6)$$

$$l_{\text{charge},i,t,s} = (1 - \eta_{\text{charge},i}) P_{\text{BES},i,t,s}^- \quad (7)$$

$$l_{\text{discharge},i,t,s} = \left(\frac{1}{\eta_{\text{discharge},i}} - 1 \right) P_{\text{BES},i,t,s}^+ \quad (8)$$

$$E_{i,t,s} = E_{i,t-1,s} - (P_{\text{BES},i,t,s} + l_{\text{charge},i,t,s} + l_{\text{discharge},i,t,s}) \Delta t \quad (9)$$

where for each scenario s , $P_{\text{BES},i,t,s}$ represent the energy output of BES of prosumer i at period t ; $P_{\text{BES},i,t,s}^+$ and $P_{\text{BES},i,t,s}^-$ are non-negative variables defined for charging and discharging power of BES; $\eta_{\text{charge},i}$ and $\eta_{\text{discharge},i}$ represent the efficiency of the charging and discharging process of BES, respectively. The power losses that are wasted in the process of charging and discharging the BES are calculated in (7) and (8) and included in the objective function to be penalized. Equation (9) shows the state of charge (SOC) of the battery in each period, which is determined based on the amount of the state of charge at the previous period and the amount of charging or discharging performed in the current period. Also, it is assumed the energy level of BES matches the battery rating at both the beginning of the first interval and the end of the optimization horizon.

The amount of energy exchange for each prosumer is given as follows.

$$P_{\text{user},i,t,s} = P_{\text{li},t,s} - P_{\text{gi},t,s} - P_{\text{BES},i,t,s} \quad (10)$$

$$Q_{\text{user},i,t,s} = Q_{\text{li},t,s} - Q_{\text{gi},t,s} - Q_{\text{BES},i,t,s} \quad (11)$$

where $P_{\text{user},i,t,s}$ and $Q_{\text{user},i,t,s}$ are the net active reactive power consumption of each prosumer; $P_{\text{li},t,s}$ and $Q_{\text{li},t,s}$ represent the active and reactive power of load demand for each prosumer. In this paper, it is assumed PV units and BES are operated at unity power factor ($Q_{\text{gi},t,s} = Q_{\text{BES},i,t,s} = 0$). By adding

each prosumer's contribution to the network losses, the power exchanged with the grid and other prosumers is determined.

$$P_{i,t,s} = P_{\text{user},i,t,s} + P_{\text{loss},i,t,s} \quad (12)$$

$$Q_{i,t,s} = Q_{\text{user},i,t,s} + Q_{\text{loss},i,t,s} \quad (13)$$

where $P_{\text{loss},i,t,s}$ and $Q_{\text{loss},i,t,s}$ represent the real and imaginary parts of losses; and $P_{i,t,s}$ and $Q_{i,t,s}$ are the power exchanged by each prosumer with the network, considering the share of power losses.

The relations and constraints of the energy exchange of a prosumer with the main network or other prosumers are defined in (14) and (15).

$$P_{i,t,s} = P_{\text{grid},i,t,s} + P_{\text{LEC},i,t,s} \quad (14)$$

$$P_{\text{grid},i,t,s} = P_{\text{buy_grid},i,t,s} - P_{\text{sell_grid},i,t,s} \quad (15)$$

where $P_{\text{grid},i,t,s}$ and $P_{\text{LEC},i,t,s}$ represent the power exchange of prosumer i with the main grid and with other prosumers at period t , respectively. The dual variables associated with (14) enable the derivation of the shadow price of $P_{\text{sell_grid},i,t,s}$ i.e., the price of the energy transaction between the prosumers of the LEC at each period. This shadow price reflects the equilibrium condition where both P_{grid} and P_{LEC} should ideally share the same price, provided that neither is null.

In the proposed model, the branch flow model known as Distflow [35] is used to represent the power flow equations of the LEC radial distribution network. The Distflow representation provides the square rms values of the terminal voltages, v_{in} and v_{out} , as well as the current for each branch, u . Each branch is associated with the user connected at its output. In this representation, each branch is characterized by a series resistance r and a series reactance x . The power losses, P_{loss} and Q_{loss} , in each branch are expressed as ru and xu , respectively. The losses in each branch are allocated to the user at the end of the branch only in the scheduling optimization procedure and price calculation. For billing purposes, a fair distribution of branch losses among all community members is needed, as the one described in [14] based on the readings of the meter at the connection with the external grid and of the users' meters. This procedure is justified by the cooperative nature of the LEC and the assumption that all the users connected to the network belong to the same community.

III. PROPOSED METHODOLOGY

The problem of simultaneous scheduling of energy and storage systems in the energy community is a problem with integer variables. Although the presence of a nonlinear component, which can make convergence difficult, has been addressed by using linearized equations, the existence of binary variables and the nature of the mixed integer programming (MIP) may be computationally expensive. Therefore, a method is adopted to convert the MIP problem into a series of simpler sub-problems. In the following, there is a description of the scenario generation and the optimization solution procedure.

A. Scenario Generation Based on Clustering

The simultaneous consideration of uncertain parameters

such as load demand and renewable generation output leads to the production of many scenarios. This large number of scenarios results in longer problem-solving time and increased complexity when trying to achieve convergence. A clustering algorithm is therefore needed. For the results shown in this paper, the basic version of the k -means algorithm available in off-the-shelf commercial or open-source optimization software [36] has been adopted. Intuitively what k -means does is that given the number k of clusters the algorithm tries to group into k clusters based on how close (similar) the scenarios are to each other. The choice of the number of clusters k is carried out by applying the elbow method [37]. The elbow method (starting with just one cluster) gradually increases the number of clusters until going further does not add much value (in terms of the amount of decrease in the minimum sum of squares error).

B. Determining the Values of Discrete Variables

As mentioned, the optimization problem of decentralized energy and reserve scheduling is a mixed integer programming one. This paper employs a two-fold iterative algorithm to address this challenge. The first layer assumes fixed discrete values and handles the rest as conic programming (CP) using the privacy-preserving distributed Limited-memory BFGS (L-BFGS) [38] approach. Subsequently, the second layer determines the values of the integer variables. In the final iteration, once the integer variables are determined, the first layer computes the dual variable of (14) and the corresponding shadow prices for $P_{\text{LEC},i,t,s}$.

In the second layer, a multi-start local search (MSLS) approach is implemented. Initially, integer variables are chosen as random values, forming a vector of all integers X_0 . Then, a random component of the vector X_0 is modified by either incrementing or decrementing it by 1, creating $\hat{X}_0$. If $\hat{X}_0$ results in a better objective value than X_0 , then $X_1 = \hat{X}_0$; otherwise $X_1 = X_0$. The process is then repeated for X_1 , picking another entry at random and randomly deciding to increment or decrement it by 1, resulting in $\hat{X}_1$. If the objective function by using $\hat{X}_1$ is better than X_1 , then $X_2 = \hat{X}_1$; otherwise $X_2 = X_1$. This process continues until no further improvement can be achieved.

This MSLS approach is simpler and more straightforward than other approaches, e.g., the Variable neighborhood search (VNS) approach adopted in [29].

C. The Solution Approach

Let x be the vector of all decision variables, L represents the feasible set of all linear constraints ($L = \{x | Ax = b, lb \leq x \leq ub\}$), and C represents the feasible set of the second-order cone. The objective is to optimize the function $\min\{c^T x | x \in L \cap C\}$. The proposed method employs alternating projection to optimize this function, as described below.

The process begins with an initial guess $\bar{x}$ and the linearly constrained quadratic programming problem is solved to find $x_0 = \arg \min \{c^T x + \frac{\gamma}{2} \|x - \bar{x}\|_2^2 | x \in L\}$. Next, x_0 is projected onto the cone C using the method presented in [34]. Denoting this projection as $\hat{x}_1 = \text{Proj}_C(x_0)$, the problem is solved to find $x_1 = \arg \min \{c^T x + \frac{\gamma}{2} \|x - \hat{x}_1\|_2^2 | x \in L\}$, and

this process continues iteratively while the total amount of infeasibilities for the conic constraints $P_{ini,t}^2 + Q_{ini,t}^2 \leq v_{ini,t} \cdot u_{i,t}$ is greater than a threshold.

The total amount of infeasibilities $I(x)$ is defined as:

$$I(x) = \sum \max\{0, (P_{ini,t})^2 + (Q_{ini,t})^2 - v_{ini,t} u_{i,t}\} \quad (16)$$

The overall algorithm procedure is described in Algorithm 1.

Algorithm 1: Alternating conic projection method

- 1 Start from an initial point $\bar{x}$ (choose $\bar{x} = 0$ if none is given)
 - 2 Set $\gamma = \gamma_0$ and $i = 0$
 - 3 Solve $x_0 = \arg \min\{c^T x + \frac{\gamma}{2} \|x - \bar{x}\|_2^2 | x \in L\}$
 - 4 **while** $I(x_i)$ is greater than a threshold **do**
 - 5 $\hat{x}_{i+1} = Proj_C(x_i)$
 - 6 $i = i + 1$
 - 7 Solve $x_i = \arg \min\{c^T x + \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 | x \in L\}$
 - 8 $\gamma = \gamma \times \gamma_f$
 - 9 **end**
-

The penalty parameter is started with an initial value γ (step 2) and gradually increases by multiplying it by $\gamma_f > 0$ (step 4.4) to obtain very accurate results for the conic programming problem.

Limited-memory BFGS (LBFGS) [38] is applied to solve the Lagrangian dual of quadratically perturbed LP $\min\{c^T x + \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 | x \in L\}$ (step 4.3). The advantage of this method is the parallelized privacy-preserving method runs with a lower number of communications [39]. The process of solving $\min\{c^T x + \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 | x \in L\}$ in each iteration is elaborated as follows.

To ensure consistent notations, we rephrase the problem as $\max\{-c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 | x \in L\}$. The Lagrangian dual of this problem is expressed as:

$$\min_{\lambda} \max_{lb \leq x \leq ub} \left\{ -c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - \lambda^T (Ax - b) \right\} \quad (17)$$

The maximum of $-c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - \lambda^T (Ax - b)$ for a fixed λ is achieved when $\frac{\partial}{\partial x} (-c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - \lambda^T (Ax - b)) = 0$, which leads to $-c - \gamma(x - \hat{x}_i) - A^T \lambda = 0$. Solving for x , we get

$$x(\lambda) = proj_{lb \leq x \leq ub} \left(-\frac{A^T \lambda + c - \gamma \hat{x}_i}{\gamma} \right) \quad (18)$$

Let's define $f(\lambda) = \max_{lb \leq x \leq ub} \{-c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - \lambda^T (Ax - b)\}$. We can then assume:

$$f(\lambda) = -c^T x(\lambda) - \frac{\gamma}{2} \|x(\lambda) - \hat{x}_i\|_2^2 - \lambda^T (Ax(\lambda) - b) \quad (19)$$

The rewritten problem (1) becomes:

$$\min_{\lambda} f(\lambda) = \min_{\lambda} \left\{ -c^T x(\lambda) - \frac{\gamma}{2} \|x(\lambda) - \hat{x}_i\|_2^2 - \lambda^T (Ax(\lambda) - b) \right\} \quad (20)$$

To solve the above problem, we start with an initial guess λ_0 and iterate as follows:

$$\lambda_{k+1} = \lambda_k + t_k d_k, \quad k = 0, 1, 2, \dots \quad (21)$$

Here, the basic gradient descent uses $d_k = -g_k$ where g_k is the gradient of $f(\lambda_k)$:

$$g_k = \nabla f(\lambda_k) = -(Ax(\lambda_k) - b) \quad (22)$$

However, d_k can be using a more sophisticated approach than just g_k . For example, the Newton method utilizes $d_k = -[\nabla^2 f(\lambda_k)]^{-1} \nabla f(\lambda_k)$ [7], where $\nabla^2 f(\lambda_k)$ is the second derivative (Hessian) of $f(\lambda)$ with respect to λ at λ_k . Computing $[\nabla^2 f(\lambda_k)]^{-1}$ is computationally expensive and memory-intensive, making it impractical for some applications. To address this issue, a middle-ground approach can be adopted, such as the Accelerated Gradient Descent (AGD) presented in [40], [41], which can be implemented in a scalable and distributed privacy-preserving fashion, as shown in [30]. AGD involves a different iteration step compared to the standard gradient descent:

$$\lambda_{k+1} = \zeta_k - t_k \nabla f(\zeta_k) \quad (23)$$

where ζ_k is a specific linear combination of the past guesses $\lambda_0, \dots, \lambda_k$. This auxiliary sequence captures curvature characteristics similar to Newton's method. For further details on AGD and its implementation in a privacy-preserving parallel fashion, refer to [30].

Another method is to approximate $D^{-1} \approx [\nabla^2 f(\lambda_k)]^{-1}$ and then use $d_k = -D^{-1} g_k$ as the search direction in (6). One successful method for this approximation is Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [42], which uses past k iteration increments

$$p_1 = \lambda_{n-k+1} - \lambda_{n-k}, \dots, p_k = \lambda_n - \lambda_{n-1} \quad (24)$$

along with the corresponding past k gradient increments:

$$\begin{aligned} q_1 &= \nabla f(\lambda_{n-k+1}) - \nabla f(\lambda_{n-k}), \dots, \\ q_k &= \nabla f(\lambda_n) - \nabla f(\lambda_{n-1}) \end{aligned} \quad (25)$$

Although the BFGS method is computationally more efficient than Newton's method, it still poses practical challenges for very large-scale problems due to the substantial memory required to store matrix D^{-1} . In contrast, L-BFGS [38] directly approximates $D^{-1}g$ instead of first approximating D^{-1} and then $D^{-1}g$.

The core of L-BFGS is the "two-loop recursion" [38], which takes the gradient g , as well as $p_1, \dots, p_n$ and $q_1, \dots, q_k$ as inputs, and outputs the search direction d (see Algorithm 2).

In (6), the determination of t_k (the step size at step k) is crucial. It can be fixed, as proposed in [40], or chosen adaptively, for example, by backtracking line search [42, Algorithm 9.2].

To solve (20) and iterate to the next candidate for λ , we use the following equation:

$$\lambda = \lambda + td \quad (26)$$

However, finding the near-optimal t requires computing $f(\lambda + td)$ multiple times, leading to significant computational and communication overhead in our case of parallel privacy-preserving computation. To address this, from (18) we drive the following relationship:

$$x(\lambda + td) = proj_{lb \leq x \leq ub} \left(-\frac{A^T(\lambda + td) + c - \gamma \hat{x}_i}{\gamma} \right)$$

Algorithm 2: Two-Loop Recursion Subroutine for LBFGS

Input: $g, q_1, \dots, q_k, p_1, \dots, p_k$
Output: search direction t

```

1  $U = g$ 
2 for  $j = k$  to 1 do
3    $\theta_j = \frac{p_j^T U}{p_j^T q_j}$ 
4    $U = U - \theta_j q_j$ 
5 end
6  $r = U$ 
7 for  $j = 1$  to  $k$  do
8    $\eta_j = \frac{q_j^T r}{p_j^T q_j}$ 
9    $r = r + p_j(\theta_j - \eta_j)$ 
10 end
11  $d = -r$ 

```

$$= \text{proj}_{lb \leq x \leq ub} \left(-\frac{A^T \lambda + t A^T d + c - \gamma \hat{x}_i}{\gamma} \right) \quad (27)$$

Similarly,

$$\begin{aligned}
f(\lambda + td) &= -c^T x(\lambda + td) - \frac{\gamma}{2} \|x(\lambda + td) - \hat{x}_i\|_2^2 \\
&\quad - (\lambda + td)^T (Ax(\lambda + td) - b) \\
&= -c^T x(\lambda + td) - \frac{\gamma}{2} \|x(\lambda + td) - \hat{x}_i\|_2^2 \\
&\quad - x(\lambda + td)^T (A^T \lambda + t A^T d) + (\lambda + td)^T b
\end{aligned} \quad (28)$$

This implies that once we compute $A^T \lambda$ and $A^T d$, we can reuse them to compute $f(\lambda + td)$ for various t values without the need for extra matrix-vector multiplications or communication. By using (28), our modification of the backtracking line search is as follows:

Algorithm 3: Modified backtracking line search

Input: $c, A, b, \hat{x}_i, \gamma, \alpha, \beta, \lambda, lb, ub, d$ and g
Output: step-size t

```

1  $A_d = Ad, A_\lambda = A^T \lambda, t = 1$ 
2  $\lambda' = \lambda + td, x = \text{proj}_{lb \leq x \leq ub} (-(A_\lambda + c - \gamma \hat{x}_i)/\gamma)$ 
3  $x' = \text{proj}_{lb \leq x \leq ub} (-(A_\lambda + t A_d + c - \gamma \hat{x}_i)/\gamma)$ 
4  $f(\lambda) = -c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - x^T A_\lambda + \lambda^T b$ 
5  $f(\lambda') = -c^T x' - \frac{\gamma}{2} \|x' - \hat{x}_i\|_2^2 - x'^T (A_\lambda + t A_d) + \lambda'^T b$ 
6 while  $f(\lambda') > f(\lambda) + \alpha t g^T d$  do
7    $t = t\beta$ 
8    $\lambda' = \lambda + td$ 
9    $x' = \text{proj}_{lb \leq x \leq ub} \left( -\frac{A_\lambda + t A_d + c - \gamma \hat{x}_i}{\gamma} \right)$ 
10   $f(\lambda') = -c^T x' - \frac{\gamma}{2} \|x' - \hat{x}_i\|_2^2 - x'^T (A_\lambda + t A_d) + \lambda'^T b$ 
11 end

```

However, compared to the L-BFGS presented in [38], our implementation differs slightly. First note that from (21) and (24) we arrive at:

$$p_{n+1} = t_n d_n \quad (29)$$

Now, the overall L-BFGS algorithm can be outlined as follows:

Algorithm 4: LBFGS algorithm for quadratically penalized LP

Input: $c, A, b, \hat{x}_i, \gamma, \alpha, \beta, \lambda, lb, ub, k_{\max}$ and δ
Output: x and λ

```

1  $A_\lambda = A^T \lambda$ 
2  $x = \text{proj}_{lb \leq x \leq ub} (-(A_\lambda + c - \gamma \hat{x}_i)/\gamma)$ 
3  $f(\lambda) = -c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - x^T A_\lambda + \lambda^T b$ 
4  $g = -(Ax - b), d = -g$ 
5 do Algorithm 3 to find good step-size  $t$ 
6  $\lambda = \lambda + td$ 
7  $k = 1, T = \text{True}$ 
8 while  $T$  do
9    $p_k = td$ 
10   $A_\lambda = A^T \lambda$ 
11   $x = \text{proj}_{lb \leq x \leq ub} (-(A_\lambda + c - \gamma \hat{x}_i)/\gamma)$ 
12   $f(\lambda) = -c^T x - \frac{\gamma}{2} \|x - \hat{x}_i\|_2^2 - x^T A_\lambda + \lambda^T b$ 
13   $g_{\text{old}} = g$ 
14   $g = -(Ax - b)$ 
15  if  $\|g\| < \delta \|b\|$  then
16     $T = \text{False}$ 
17  end
18   $q_k = g - g_{\text{old}}$ 
19  do Algorithm 2 to determine new search
    direction  $d$ 
20  do Algorithm 3 to find good step-size  $t$ 
21   $k = k + 1$ 
22  if  $k > k_{\max}$  then
23     $k = 1$ 
24  end
25 end

```

Steps in Algorithms 1–4 can be implemented in a distributed privacy-preserving manner, which is advantageous.

This process is iterated until no further improvement is possible. This entire procedure is repeated multiple times, each time with random initialization, and the best result among all iterations is selected. Compared to VNS as proposed in [29], this MSLS approach is easier to implement.

IV. CASE STUDY

The method developed in this paper was implemented using MATLAB R2022b on a computer with an Intel(R) Core(TM) i7-10700 CPU @ 2.90 GHz and 16 GB of RAM.

The proposed method has been tested on an 84-bus distribution network comprising eleven feeders with an 11.4 kV rated voltage [44]. The single-line diagram of the network is illustrated in Fig. 1. For energy and reserve scheduling purposes, a one-day horizon consisting of 96 15-minute periods was taken into consideration. Prosumers are connected to all buses and equipped with PV-battery systems to feed their internal loads or inject excess energy into the network. Battery-type energy storage systems have capacities between 20 kWh and 200 kWh. The battery capacity to rated power ratio was

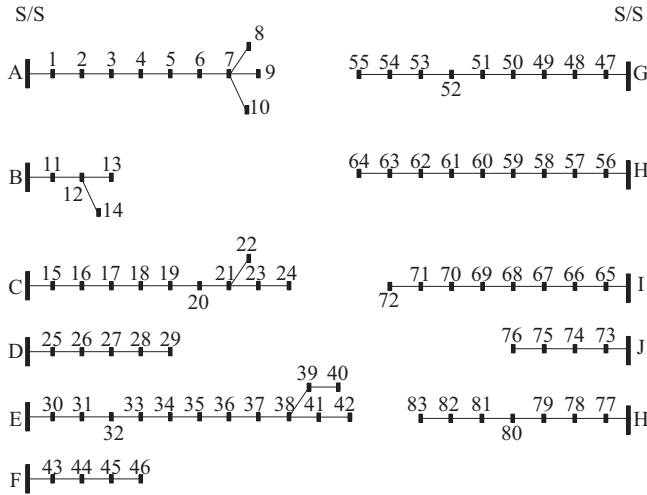


Fig. 1. 84-bus radial distribution test system.

assumed to be equal to 1 hour. $P_{\text{sell_grid}}$ has been assumed to be always zero and only up-regulation is considered to simplify the analysis.

To evaluate the effectiveness of the proposed method, three cases were compared: Case 1, where there are no energy exchanges among prosumers; Case 2, where prosumers can exchange energy within the entire LEC (connected feeders); and Case 3, where direct exchanges are allowed only among prosumers belonging to the same feeder, and exchanges of power between the community's prosumers do not result in any power flow on the grid.

The total load demand and photovoltaic power generation profiles of the test network are shown in Figs. 2 and 3. The amount of energy consumed by the LEC in 24 hours is equal to 127 MWh, while PV resources deliver energy equal to 73 MWh in the same period. Moreover, the total capacity of batteries in the network is 7.26 MWh. Therefore, in percentage terms, PV generations cover 57.5% of the load and batteries cover 9.9% of the energy produced by PVs.

At first, 100 scenarios for load demand and 100 scenarios for photovoltaic production were created by the Monte Carlo sampling method of the normal and beta probability distribution functions, respectively [45]. Then, assuming no correlation between these two variables, by the scenario tree method, joint scenarios of load and PV generation were formed.

As mentioned, the energy produced by prosumers should be primarily consumed by the community participants or to be used for storage in the batteries. In this case, we do not consider the direct sale of energy to the network, and prosumers can only purchase from the network and receive remunerations through the sale of ancillary services from 2 p.m. (or interval 56) until 6 p.m. (or interval 72) since this period is identified by the VNS method. The hourly price of buying energy from the main grid is shown in Fig. 4. Also, the hourly energy prices corresponding to energy exchanges between community prosumers for case 2 are also shown in Fig. 4. As shown, in the intervals between 1–27 and 73–96, the price of energy exchange between prosumers is equal to

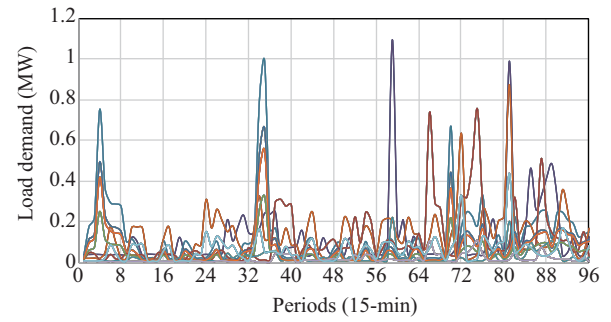


Fig. 2. Profiles of load demand for each prosumer.

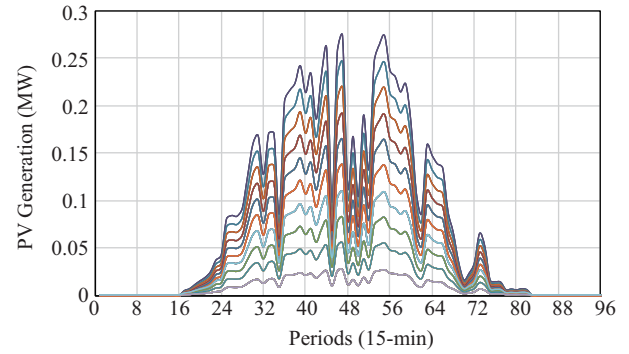


Fig. 3. Profiles and PV generation for each prosumer.

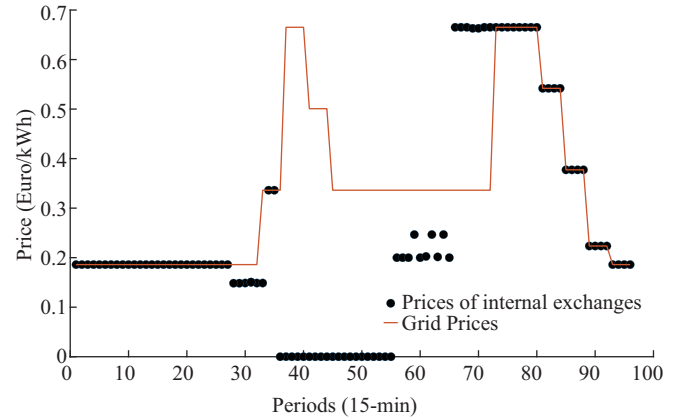


Fig. 4. Energy prices with the utility grid and the energy prices of exchanges between prosumers.

the price of buying energy from the main grid. However, in the period between 66–72, due to the flexibility provision, the price of energy exchange between prosumers is higher than the price of the grid.

The prices of energy transactions between prosumers in case 2 vary depending on the PV production and flexibility provision. From period 36 to period 55, due to the presence of excess solar power, PV energy is stored in the prosumers' batteries. Therefore, the exchange price of energy reaches its minimum level. From period 56 to 72, the LEC receives a remuneration (200 euro/MWh) for providing ancillary services to the grid. This leads to an increase in the prices of exchanges between prosumers from period 66 to 72, due to the low or

zero energy produced by PV systems, and a decrease in the prices of exchanges between prosumers from period 56 to 66, due to the high energy produced by the PV systems.

The exchange prices (shadow prices) are influenced not only by the cost of purchasing energy from the main grid but also by the compensation received for providing the flexibility service and by the production of energy by PV systems. Consequently, during these intervals, the energy exchange price tends to be higher than the price of energy obtained directly from the grid. As shown in Fig. 4, due to an excess of energy produced by the PV systems, the energy price of exchanges between prosumers is equal to zero in the intervals from period 36 to 55.

As can be deduced from Fig. 5, the price of energy in bilateral exchanges between prosumers in the case of separated feeders (i.e. case 3) is not the same at different buses. This shows that in the case of separated feeders, the prices of energy exchange between prosumers do not depend only on the profile of PV generation and load consumption in the entire network. In other words, the location of the PV and battery system in the network affects the price of energy exchange between prosumers in such a way that energy is provided at a lower price for the P2P energy transaction for the buses close to the location of the PVs/batteries.

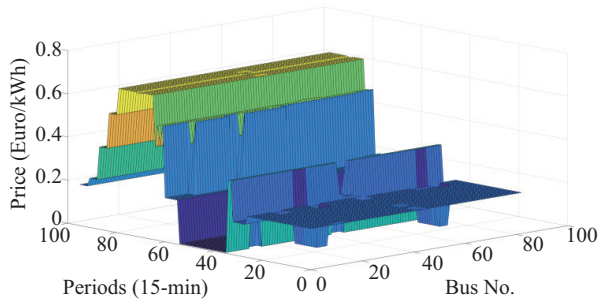


Fig. 5. The price of energy exchange between prosumers in Case 3 (separated feeders)

The values of the objective function and other indices in three different cases are presented in Table I. The sequential computing procedure requires less than 8 seconds in terms of CPU time. The baseline and up regulation values represent the objective functions without and with up regulation, respectively. Prosumers' income from providing up regulation is shown in the second row, and the total costs for the baseline and up-regulation cases are shown in the third row. In the last row, energy losses in both cases are presented. When up regulation is considered, the value of the objective function improves compared to the baseline. In all cases, implementing up regulation results in reduced energy losses and increased

energy costs, which are always compensated by the remuneration received for regulation. Therefore, the implementation of up-regulations is always convenient. Even though there is an increase in the energy costs of approximately 597 euros in the case of the LEC with joint feeders, the objective function decreases by around 830 euros. The same trend is also confirmed in the case of the LEC with separated feeders.

The benefits of implementing the LEC, compared to the case without the LEC, are evident. In the case of up-regulation, the objective function decreases from 25,238 euros to 23,196 euros with joint feeders and to 23,259 euros for the LEC with separated feeders. This decrease is due to both the remuneration received for up-regulation and a reduction in energy costs compared to the case without the LEC. The decrease in the objective function is lower in the case of separated feeders compared with joint feeders due to an increase in energy costs resulting from reduced internal energy exchanges.

The results show that the total cost of energy decreases when prosumers are allowed to engage in P2P energy transactions. For example, in comparison to case 1, the total cost of energy in cases 2 and 3 decreases by 1,899 euros and 1,782 euros, respectively. In terms of energy losses, the activation of P2P energy transactions causes a reduction in network energy losses. This reduction in energy losses is more significant in case 3, where the feeders are considered separately. This separation reduces the current between the feeders and prevents the flow of power between them.

Figure 6 illustrates how energy exchanges between prosumers result in a reduction of energy transactions with the utility grid. Prosumers with a battery benefit more from energy exchanges due to the flexibility offered by the optimal use of their battery systems.

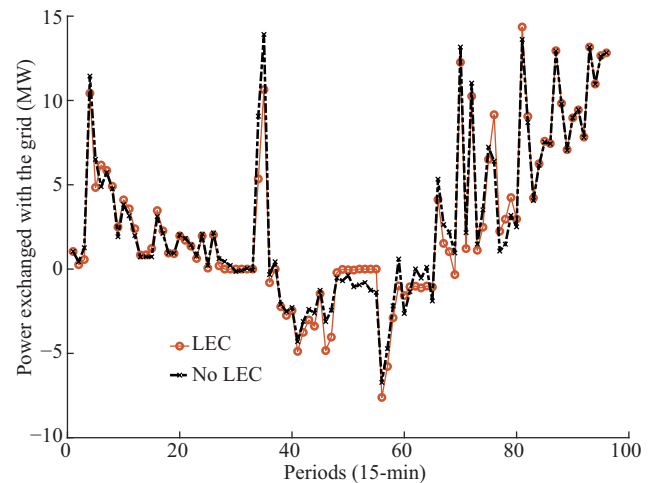


Fig. 6. Energy exchanged with the main grid in cases 1 and 2.

TABLE I
THE VALUES OF THE OBJECTIVE FUNCTION, REMUNERATION, COST OF ENERGY AND ENERGY LOSSES IN THREE CASES

Description	Case 1 No LEC		Case 2 LEC		Case 3 LEC with Separated Feeders	
	Baseline	Up regulation	Baseline	Up regulation	Baseline	Up regulation
The objective function (euro)	25,763	25,238	24,026	23,196	24,120	23,259
Remuneration (euro)	0	1,248	0	1,427	0	1,481
Cost for energy (euro)	25,763	26,522	24,026	24,623	24,120	24,740
Energy losses (MWh)	8.40	7.24	8.22	7.08	8.51	7.03

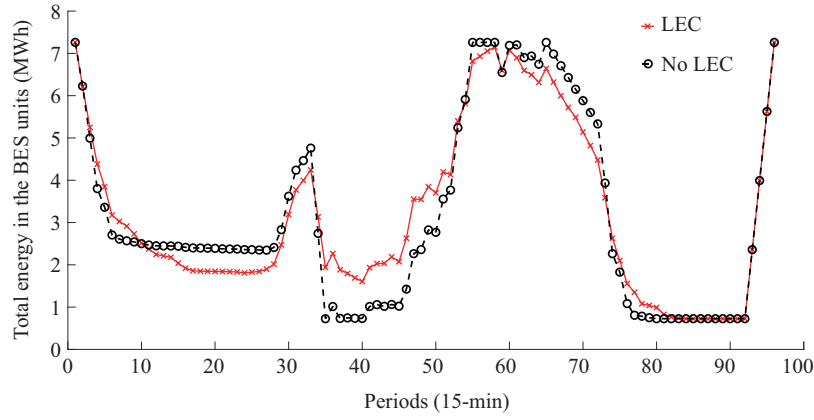


Fig. 7. Total energy in the batteries with energy exchanges among prosumers (case 2) and without energy exchanges among prosumers (case 1).

As shown in Figs. 6 and 7, from period 48 to 55, the energy exported to the grid is almost zero in the case of energy exchanges inside the LEC. Instead, in the case without the LEC, energy is exported to the main grid. There is this different behavior because, during the interval from period 48 to 55, energy exchanges within the LEC allow for the utilization of more energy produced by the PV systems in the LEC itself, rather than injecting it into the grid. The presence of P2P energy exchanges inside the LEC and their increase during this interval is also confirmed in Fig. 8.

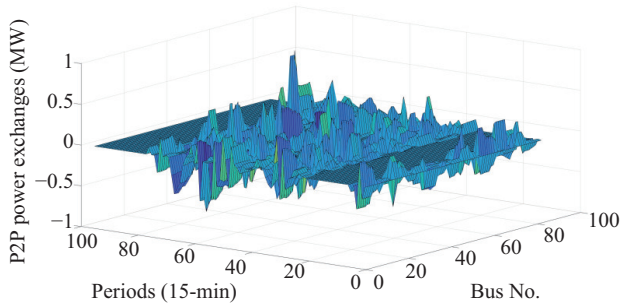


Fig. 8. P2P energy exchange among prosumers with connected feeders.

The total energy stored in batteries is shown in Fig. 7 for cases 1 and 2. From period 56 to 72, the discharge of energy from the batteries is due to the provision of grid flexibility, particularly in terms of up-regulation, to the system operator. This valuable energy flexibility service comes with an associated remuneration.

The optimal use of batteries also facilitates energy exchanges among prosumers during the day and improves the utilization of the energy generated by PV systems, leading to an increase in the self-consumption of energy within the LEC.

When considering computational complexity and scalability, it is worth noting that the considered optimization problem is related to day-ahead operational planning, so stringent computational time requirements are not essential. Given the importance of preserving user privacy, our comparative analysis in terms of computational complexity and scalability primarily focuses on decentralized, privacy preserving algorithms capable of efficiently handling large-scale problems, including those involving second-order cones. To address this,

we conducted several additional simulations and compared them with the methods presented in [4] and [30].

It is worth noting that both methods proposed in [4] and [30] proved to be more efficient compared to centralized optimization and the Splitting Conic Solver (SCS) ADMM in terms of privacy preservation, scalability, and computational performance (refer to [4] for more details). Test results demonstrate that the new method based on L-BFGS is not only significantly faster than the methods presented in [4] and [30] but also requires fewer communications and achieves accurate results with a lower number of iterations compared to the methods adopted in [4] and [34]. In particular, for the considered problem, the number of communications for the methods described in [4] and [30] is double that of LBFGS. Additionally, adopting a warm start, where the initial solution is available from the day before optimization, results in a significantly lower number of communications.

V. CONCLUSION

The paper presents an innovative iterative two-stage optimization approach for energy and reserve scheduling within an LEC, considering network constraints. By implementing the L-BFGS method, the MIP problem is effectively converted into a more manageable CP problem. This transformation significantly reduces computational complexity, rendering the proposed approach easily scalable and adaptable for the implementation on microcontrollers, thanks to its reliance on simple matrix-vector multiplication.

An essential characteristic of the proposed approach is its inclusion of LEC participants in providing ancillary reserve services to the utility grid. This incentivizes participants and concurrently lowers the total operating costs of the LEC.

By providing a simpler and more accessible solution method, the proposed approach has the potential to improve the efficiency and scalability of the simultaneous energy and reserve scheduling of an LEC.

For the practical implementation of a real energy community, the deployment of distributed intelligent devices based on Information and Communication Technology (ICT) is indispensable. We assume that each prosumer is equipped with a home device gateway, a smart meter, and an energy management system (EMS) [10]. The management system platform

presented in [10] can be used for information exchange among prosumers and for managing energy storage batteries.

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